STL

STRUCTURAL · QUICK REFERENCE 03-1

Beam Shear & Moment - Diagrams, Peaks & Deflection

Reactions, shear, moment and deflection for single-span beams, and the chain every case comes from. Span (L) · Uniform load (w) · Point load (P) · Reaction (R) · Shear (V) · Moment (M) · Deflection (δ) · Moment of inertia (I)

FORMULA

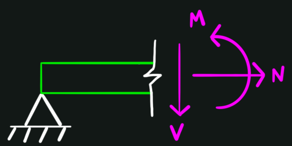
$$V(x) = \Sigma \text{ forces to the left} \qquad M(x) = \int V\,dx$$

$$\Sigma F_y = 0,\ \Sigma M = 0 \rightarrow \text{reactions} \qquad dV/dx = -w,\ dM/dx = V$$

$$E{\cdot}I{\cdot}d^2v/dx^2 = M(x) \rightarrow \text{integrate twice to obtain } v(x)$$

Shear is obtained by integrating load, moment by integrating shear, and deflection by integrating curvature twice. Reactions come from equilibrium of the whole span; the supports set the constants of integration. Constant I, bending deformation only.

WHAT THE BEAM CARRIES AT A CUT



Cut the beam and draw the free body of one side: the removed half is replaced by a transverse shear V and a bending moment M on the cut face. Loading is purely transverse, so the axial force N is zero.

WORKED EXAMPLE: 30 FT SIMPLE SPAN, UNIFORM LOAD

Given: L = 30 ft (360 in) · w = 0.9 kip/ft service load · W12×30, I_x = 238 in⁴ · E = 29,000 ksi

1. $R = wL/2 = 0.9 \times 30 / 2 = 13.5$ kip $= V_{max}$ (each support)
2. $M_{max} = wL^2/8 = 0.9 \times 30^2 / 8 = 101.3$ kip-ft (at midspan)
3. If $L_b \le L_p \approx 5.4$ ft: $M_{allow} = F_y Z_x/1.67 = 107.5$ kip-ft (A992; > 101.3 demand)
4. $\delta_{max} = 5wL^4/(384EI) = 5 \times 0.075 \times 360^4 / (384 \times 29{,}000 \times 238) = 2.38$ in
5. $L/\delta = 360 / 2.38 = 151$ (w = 0.9 kip/ft = 0.075 kip/in)

> **M_{max} = 101.3 kip-ft, δ = L/151.** Within the simplified allowable only if $L_b \le L_p$, yet it sags 2.38 in at service load; L/240 here is 1.50 in, L/360 is 1.00 in.

PRELIMINARY STEEL CHECK (ASD)

$$M_{allow} = F_y Z_x / \Omega_b \quad (\Omega_b = 1.67) \qquad f_v = V / A_w \le 0.6 F_y C_v / \Omega_v$$

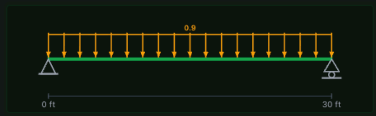
- **Compact, braced sections.** $F_y{\cdot}Z_x/1.67$ assumes $L_b \le L_p$ (AISC 360 F2). A complete check adds unbraced length, compactness and lateral-torsional buckling.

CONSIDERATIONS

- **Web shear area.** For rolled I-shapes the web generally carries most of the shear and $A_w = d{\cdot}t_w$ is commonly used; other shapes follow applicable Chapter G provisions. V / A_w is an average; actual shear stress varies across the section.
- **Where 0.40·F_y comes from.** Rolled I-shapes with C_v = 1.0 and Ω_v = 1.50 give an ASD allowable shear stress of 0.40·F_y; other sections and slender webs follow Chapter G.
- **Section modulus.** Use the applicable S_x for elastic bending stress; for unsymmetric sections such as tees and angles it can differ by extreme fiber.
- **Elastic modulus.** E = 29,000 ksi applies to steel; timber, aluminum and other materials have different E values, so update E to match the member material. Deflection is inversely proportional to E: a lower E deflects more under the same load.

WORKED EXAMPLE - DIAGRAMS

FREE BODY DIAGRAM



SHEAR (V)

MOMENT (M)

DEFLECTION (Δ)

Shear crosses zero at midspan, exactly where the moment and the deflection peak. Maximum or minimum moment occurs where shear is zero or changes sign, including across concentrated-load discontinuities.

FIELD NOTES

- **Deflection / Serviceability.** Deflection is reported independently of strength checks when I_x is available. Passing bending and shear does not mean the beam satisfies project deflection limits.
- **Splices and reinforcement.** Use the diagrams to inform splice and reinforcement locations, but consider shear, axial force, connection requirements, and constructability as well as moment.
- **What governs.** Flexure or serviceability often governs conventional rolled beams, but shear can control for short spans, heavy loads near supports, copes, and web penetrations.
- **Not covered.** Web crippling, web local yielding and block shear at the connection are separate limit states.
- **Torsion.** A transverse load through the shear center does not introduce load-eccentricity torsion. Loads eccentric to the shear center introduce torsion not shown by the V/M diagram.

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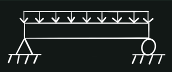
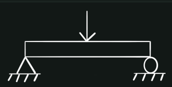
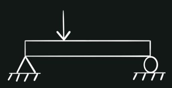
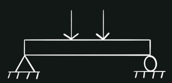
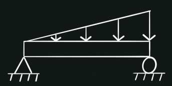
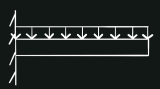
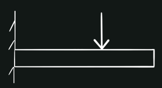
STRUCTURAL · QUICK REFERENCE 03-2

Beam Shear & Moment - Standard Cases

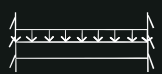
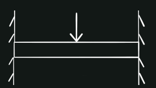
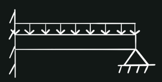
Peak shear, peak moment and maximum deflection for twelve single-span cases, for checking a result by hand. L is the span or cantilever length, w is load per unit length, P a concentrated load, and a and b are the distances from the point load to the two supports, with a + b = L.

TABLE 1 · CASES THE CALCULATOR SOLVES

BEAM	CASE	V_{MAX}	M_{MAX}	Δ_{MAX}
	**Simple span, uniform load w**	$wL/2$ **at each support**	$wL^2/8$ **at midspan**	$5 \cdot w \cdot L^4 / (384 \cdot E \cdot I)$ **at midspan**
	**Simple span, point load P at centre**	$P/2$ at each support	$PL/4$ at midspan	$P \cdot L^3 / (48 \cdot E \cdot I)$ at midspan
	**Simple span, point load P at a**	$P \cdot a/L$ at the support nearer the load	$P \cdot a \cdot b/L$ under the load	$P \cdot b \cdot (L^2 - b^2)^{3/2} / (9\sqrt{3} \cdot L \cdot E \cdot I)$ at x = $\sqrt{(L^2 - b^2)/3}$ from the support farther from the load, b the shorter distance
	**Two equal point loads P, each at a from its nearest support (simple span)**	P at each support; V = 0 between the loads	$P \cdot a$ constant between the loads	$P \cdot a \cdot (3L^2 - 4a^2) / (24 \cdot E \cdot I)$ at midspan
	**Simple span, triangular load 0 → w_0**	$w_0 L/3$ at the loaded end ($w_0 L/6$ at the unloaded end)	$w_0 L^2 / (9\sqrt{3}) = 0.0642 \cdot w_0 L^2$ at x = 0.5774·L from the unloaded end	$0.006522 \cdot w_0 \cdot L^4 / (E \cdot I)$ at x ≈ 0.5193·L from the unloaded end
	**Cantilever, uniform load w**	wL at the fixed end	$w \cdot L^2/2$ at the fixed end	$w \cdot L^4 / (8 \cdot E \cdot I)$ at the free end
	**Cantilever, point load P at free end**	P constant along the beam	$P \cdot L$ at the fixed end	$P \cdot L^3 / (3 \cdot E \cdot I)$ at the free end
	**Cantilever, point load P at a from the fixed end**	P between the fixed end and the load, zero beyond	$P \cdot a$ at the fixed end	$P \cdot a^2 \cdot (3L - a) / (6 \cdot E \cdot I)$ at the free end

Determinate spans: pin + roller, or a single fixed end. Row 1 is the worked example on sheet 03-1. Consistent inch units: L in in, P in kip, w in kip/in, I in in^4 and E in ksi give δ in inches.

TABLE 2 · REFERENCE CASES NOT SOLVED BY THE CALCULATOR

BEAM	CASE	V_{MAX}	M_{MAX}	Δ_{MAX}
	**Fixed-fixed, uniform load w**	$wL/2$ at each support	$wL^2/12$ at each end ($wL^2/24$ at midspan)	$w \cdot L^4 / (384 \cdot E \cdot I)$ at midspan
	**Fixed-fixed, point load P at centre**	$P/2$ at each support	$PL/8$ at both ends and at midspan	$P \cdot L^3 / (192 \cdot E \cdot I)$ at midspan
	**Propped cantilever (fixed one end, pinned the other), uniform load w**	$5wL/8$ at the fixed end ($3wL/8$ at the pin)	$wL^2/8$ at the fixed end; peak sagging moment $9wL^2/128$ at x = 3L/8 from the pin	$\approx 0.00542 \cdot w \cdot L^4 / (E \cdot I)$ at x ≈ 0.578·L from the pinned end, not midspan
	**Cantilever, moment M applied at the free end**	0 zero throughout	M constant along the beam	$M \cdot L^2 / (2 \cdot E \cdot I)$ at the free end

Rows 9 to 11 are statically indeterminate; row 12 needs an applied moment, which is not one of the load types the beam builder offers.